1. Se Th. 6/752 i boken: Calculus – A Complete Course eller kurs-anteckningarna.

2.

a) $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ existerar <u>inte</u>

därför att om $\underline{x=0}$ $\quad \lim\limits_{(x,y)\to(0,0)} f(x,y) = 0$

och om $\underline{x=y}$ $\quad \lim\limits_{(x,y)\to(0,0)} f(x,y) =$

$$= \lim_{x\to 0} \frac{x^4}{x^4+x^4+x^4} = \frac{1}{3}.$$

alltså f kan inte vara kontinuerlig i punkten $(0,0)$.

b) $$f'_x(0,0) = \lim_{x\to 0} \frac{f(x,0)-f(0,0)}{x-0} = \lim_{x\to 0} \frac{0-0}{x} = 0$$

$$f'_y(0,0) = \lim_{y\to 0} \frac{f(0,y)-f(0,0)}{y-0} = \lim_{y\to 0} \frac{0-0}{y} = 0$$

c) $$f'_x(x,y) = y \cdot \frac{3x^2(x^4+x^2y^2+y^4) - x^3(4x^3+2xy^2)}{(x^4+x^2y^2+y^4)^2}$$

$$= \frac{y(3x^6+3x^4y^2+3x^2y^4-4x^6-2x^4y^2)}{(x^4+x^2y^2+y^4)^2}$$

$$= \frac{y(-x^6+x^4y^2+3x^2y^4)}{(x^4+x^2y^2+y^4)^2} = \frac{x^2y(-x^4+x^2y^2+3y^4)}{} \quad \forall (x,y) \neq (0,0)$$

$$f'_x(1,1) = \frac{1(-1+1+3)}{(1+1+1)^2} = \frac{3}{9} = \frac{1}{3}.$$

3.

f diff i $(a,b) \iff f(x,y) = f(a,b) + f'_x(a,b)(x-a) + f'_y(a,b)(y-b) + \omega(x,y)\cdot\sqrt{(x-a)^2+(y-b)^2}$

där $\omega(x,y) \to 0$ når $(x,y) \to (a,b)$.

$f(0,0) = 0$

$$f'_x(0,0) = \lim_{x\to 0} \frac{0-0}{x} = 0$$

$f'_y(0,0) = 0.$

$$\Rightarrow \omega(x,y) = \frac{\sqrt{|x\,y|}}{\sqrt{x^2+y^2}}$$

$\lim\limits_{(x,y)\to(0,0)} \omega(x,y)$ existerar <u>INTE</u> ty

om $x=0$ $\quad \lim\limits_{(x,y)\to(0,0)} \omega(x,y) = 0$ och

om $x=y>0$ $\quad \lim\limits_{\substack{x\to 0\\ y\to 0}} \omega(x,y) = \lim\limits_{(x,y)\to(0,0)} \frac{x}{\sqrt{2}\,x} = \frac{1}{\sqrt{2}}$,

dvs. funktionen är <u>inte</u> differentierbar.

4. $f'_x(x,y) = -6x - \frac{6x}{1+x^2+y^2}$

$f'_y(x,y) = 2y - \frac{6y}{1+x^2+y^2}$

$\Rightarrow$ $(0,0)$ är den enda stationära punkten

$\boxed{f(0,0) = 2}$ Funktionen är definierad på ett slutet och begränsat området, alltså abs. min och abs. max existera

$L(x,y,\lambda) = 2 - 3x^2 + y^2 - 3\ln(1+x^2+y^2) + \lambda(x^2+y^2-1)$

$$\begin{cases} \frac{\partial L}{\partial x} = -6x - \frac{6x}{1+x^2+y^2} - 2\lambda x \\ \frac{\partial L}{\partial y} = 2y - \frac{6y}{1+x^2+y^2} - 2\lambda y \\ \frac{\partial L}{\partial \lambda} = x^2+y^2-1 \end{cases}$$

Stationära punkter till Lagrange-funktionen:

$$\begin{cases} (0, 1, \frac{1}{2}) \\ (0, -1, \frac{1}{2}) \\ (1, 0, -\frac{9}{2}) \\ (-1, 0, -\frac{9}{2}) \end{cases}$$

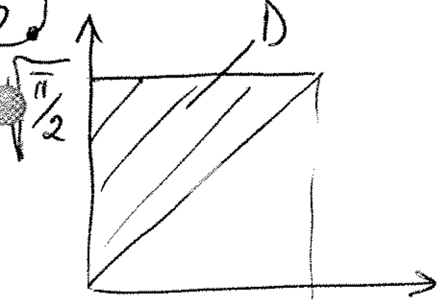
$f(0,1) = f(0,-1) = 3 - 3\ln 2$

$f(1,0) = f(-1,0) = -1 - 3\ln 2$ abs. min

$$\left.\begin{aligned} f''_{xx}(0,0) &= -12 \\ f''_{yy}(0,0) &= -5 \\ f''_{xy}(0,0) &= 0 \end{aligned}\right\} \Rightarrow (0,0) \text{ är en } \underline{\text{punkt}} \text{ av lok. max}$$

men $f(0,0) > f(0,1) \Rightarrow (0,0)$ är $\underline{\text{abs}}$ max.

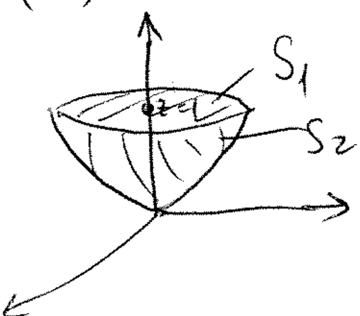
5.



$$\iint_D \cos y^2 \,dx\,dy = \int_0^{\sqrt{\frac{\pi}{2}}} \left(\int_0^{y} \cos y^2 \,dx\right) dy$$

$$= \int_0^{\sqrt{\frac{\pi}{2}}} y \cos y^2 \,dy \overset{\text{v.b.}}{=} \frac{1}{2}.$$

6. (a)



t.ex. cylindriska koordinater

$$\iiint_K z\sqrt{x^2+y^2}\,dx\,dy\,dz = \int_0^1 \left(\iint_{x^2+y^2\le z} z\sqrt{x^2+y^2}\,dx\,dy\right)dz$$

$$= \int_0^1 z\left(\int_0^{2\pi}\int_0^{\sqrt{z}} r^2\,dr\,d\theta\right)dz = \int_0^1 z\left(\int_0^{2\pi} d\theta\right)\left[\frac{r^3}{3}\right]_0^{\sqrt{z}} dz$$

$$= 2\pi \int_0^1 \frac{z^{3/2}}{3}\,dz = \cancel{\frac{2\pi}{3}} \frac{2\pi}{3} \cdot \frac{z^{\frac{3}{2}+1}}{\frac{3}{2}+1} = \frac{2\pi\cdot 2}{3\cdot \cancel{3}5}$$

$$= \frac{4\pi}{\ldots}$$

6(b) $A(S_2) = \iint_{S_2} d\sigma = \iint_{x^2+y^2 \le 1} \sqrt{1+(2x)^2+(2y)^2}\, dx\, dy \overset{\text{pol. koordinater}}{=} \frac{(5\sqrt{5}-1)\pi}{6}$.

$A(S_1) = \pi$

$\Rightarrow A(S) = A(S_1) + A(S_2) = \frac{(5\sqrt{5}+5)\pi}{6}$.

7 a) $Q(x,y) = (1+y)e^{x^2+y}$; $P(x,y) = 2xye^{x^2+y}$

$Q, P: \mathbb{R}^2 \to \mathbb{R}$

$\mathbb{R}^2$ är enkelt sammanhängande

$\vec{F}$ konservativt $\Longleftrightarrow \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

$\frac{\partial P}{\partial y} = 2x(1+y)e^{x^2+y} = \frac{\partial Q}{\partial x}$

$f = ?$

en potential så att $\nabla f = \vec{F}$

d.v.s $\begin{cases} f'_x(x,y) = P(x,y) \\ f'_y(x,y) = Q(x,y) \end{cases}$

$f'_x(x,y) = 2xy\, e^{x^2+y}$

$\Rightarrow f(x,y) = y\, e^{x^2+y} + C(y)$

$f'_y(x,y) = (1+y)e^{x^2+y} + C'(y) = (1+y)e^{x^2+y}$

$\Rightarrow C'(y) = 0 \Rightarrow C(y) = k, \quad k = 0,$

$$\Rightarrow \boxed{f(x,y) = y e^{x^2+y}} \qquad \begin{array}{l} t=0 \Rightarrow A(0,0) \\ t=\pi \Rightarrow B(\pi,2) \end{array}$$

$$\Rightarrow \int_C \vec{F}\cdot d\vec{r} = f(\pi,2) - f(0,0) = 2e^{\pi^2+2} - 0$$

$$= 2e^{\pi^2+2}.$$

(b) $$\int_C ds = \int_0^{\pi} \sqrt{(1-\cos t)^2 + (\sin t)^2}\, dt$$

$$= \sqrt{2} \int_0^{\pi} \sqrt{1-\cos t}\, dt$$

$$= \sqrt{2} \int_0^{\pi} \sqrt{2}\cdot \sin\frac{t}{2}\, dt = \sqrt{2}\cdot\sqrt{2} \left[\frac{-\cos\frac{t}{2}}{\frac{1}{2}} \right]_0^{\pi} = -4\left(\cos\frac{\pi}{2} - \cos 0\right)$$

$$= 4.$$

8. $$\iint_S \vec{F}\cdot d\vec{\sigma} = \int_0^{\pi}\int_0^{2\pi} \left(\sin u\cos v \cdot \sin^2 u \cos v + \sin u \sin v \cdot (\sin^2 u)\sin v + \cos u \cdot \sin u \cos u \right) du\, d$$

$$\vec{r}'_u \times \vec{r}'_v = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos u\cos v & \cos u \sin v & -\sin u \\ -\sin u \sin v & \sin u \cos v & 0 \end{bmatrix} = \sin^2 u \cos v\, \vec{i} - + \sin^2 u \sin v\, \vec{j} + + \sin u \cos u\, \vec{k}$$

$$= \int_0^{\pi}\int_0^{2\pi} \left(\sin^3 u \cos^2 v + \sin^3 u \sin^2 v + \sin u \cos^2 u \right) du\, dv$$

$$= \int_0^{\pi} \int_0^{2\pi} (\sin^3 u + \sin u \cos^2 u)\, du\, dv$$

$$= 2\pi \int_0^{\pi} \sin u \, du = 2\pi \left(-\cos u\right)_0^{\pi} = 2\pi \cdot 2 = 4\pi .$$