1. Se boken

2. a) $x = y$ $\quad \lim_{x\to 0} \frac{(x-x)^2}{x^2+x^2} = 0$.

$x=0$ $\quad \lim_{y\to 0} \frac{y^2}{y^2} = 1$

$\Longrightarrow$ gränsvärdet existerar inte.

b) $$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial x} = \frac{\partial z}{\partial u} + 2x\frac{\partial z}{\partial v}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y} = \frac{\partial z}{\partial u} + 2y\frac{\partial z}{\partial v}$$

$$\Rightarrow x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = x\frac{\partial z}{\partial u} + 2x^2\frac{\partial z}{\partial v} + y\frac{\partial z}{\partial u} + 2y^2\frac{\partial z}{\partial v} =$$

$$= (x+y)\frac{\partial z}{\partial u} + 2(x^2+y^2)\frac{\partial z}{\partial v}$$

$$= u\frac{\partial z}{\partial u} + 2v\frac{\partial z}{\partial v}.$$

3. $f(x,y,z) = xy + yz - 4xz$

$\nabla f = (y-4z,\ x+z,\ y-4x)$

$\nabla f(1,2,1) = (2-4,\ 1+1,\ 2-4) = (-2,2,-2)$

$\nabla f(1,2,1)$ är en normalvektor till ytan $xy+yz-4xz=0$

i punkten $(1,2,1)$

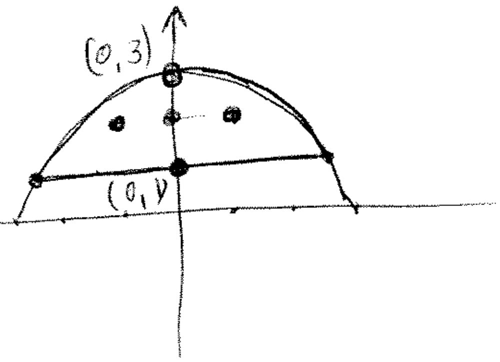
$g(x,y,z) = 3z^2 - 5x + y$ $\qquad \nabla g = (-5, 1, 6z)$

$\nabla g(1,2,1) = (-5,1,6)$ är en normalvektor till ytan $3z^2-5x+y=0$ i punkten $(1,2,1)$.

$$\cos\theta = \frac{\nabla f(1,2,1)\cdot\nabla g(1,2,1)}{|\nabla f(1,2,1)|\,|\nabla g(1,2,1)|} = \frac{(-2,2,-2)\cdot(-5,1,6)}{\sqrt{3\cdot 2^2}\cdot\sqrt{25+1+36}}$$

$$= \frac{10+2-12}{2\sqrt{3}\,\sqrt{62}} = 0 \implies \theta = \frac{\pi}{2}.$$

4.



stationära punkter:

$$\begin{cases} f_1(x,y) = 2x & 2x=0 \quad \boxed{x=0} \\ f_2(x,y) = -2y+4 & \boxed{y=2} \end{cases}$$

$(0,2)$

$f_{11}(x,y) = 2$

$f_{22}(x,y) = -2$ $\qquad f_{11}\cdot f_{22} - (f_{12})^2 = -4 < 0$

$f_{12}(x,y) = 0$ $\qquad \Rightarrow (0,2)$ är terasspunkt.

$x^2+y^2=9$ $\qquad$ max

$\underline{\underline{y=1}}$ $\qquad f(x,1) = x^2-1+4 = x^2+3 \qquad x\in[-2\sqrt{2}, 2\sqrt{2}]$ $\qquad f(-2\sqrt{2},1) = 8+... = 11$, $f(2\sqrt{2},1) = \underline{\underline{11}}$

$f' = 2x \;;\; \underline{x=0}$ $\qquad (0,1)$ min punkt

$\boxed{f(0,1) = 0-1+4 = 3}$ min

$$L(x,y,z) = x^2-y^2+4y+\lambda(x^2+y^2-9)$$

$$\begin{cases} \dfrac{\partial L}{\partial x} = 2x+2\lambda x = 0 & x(1+\lambda)=0 \Rightarrow x=0 \text{ eller } \underline{\lambda=-1} \\ \dfrac{\partial L}{\partial y} = -2y+2\lambda y+4 = 0 & 2y(-1+\lambda)+4=0 \quad -4y=-4 \quad \boxed{y=1} \\ \dfrac{\partial L}{\partial \lambda} = x^2+y^2-9 = 0 & \boxed{y=3} \qquad f(0,3) = \end{cases}$$

$x^2=8 \qquad x=\pm 2\sqrt{2} \qquad = 0-9+12 = 3.$

$(0, 3, -\frac{1}{3})$

$(\sqrt{5}, 2, -1)$

$(-\sqrt{5}, 2, -1)$

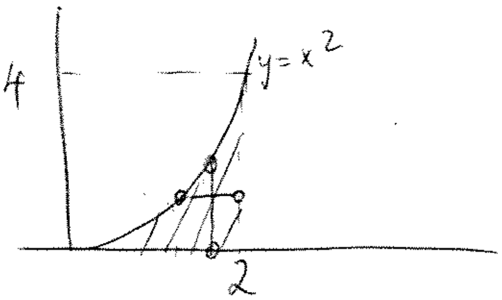
$$\cancel{f(\sqrt{5}, 2) = 5 - 4 + 4 \cdot 2 = 5 - 4 + 8 = 9.}$$

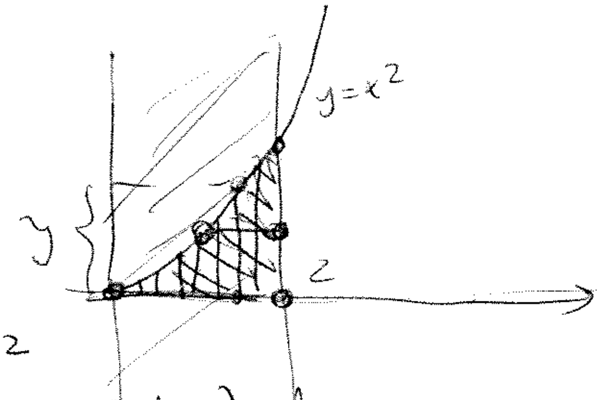
$$\cancel{f(-\sqrt{5}, 2) = 9.}$$

~~maximip.~~

maximipunkt

5.





$$\int_0^4 \left(\int_{\sqrt{y}}^2 \cos x^3 \, dx \right) dy = \int_0^2 \left(\int_0^{x^2} \cos x^3 \, dy \right) dx$$

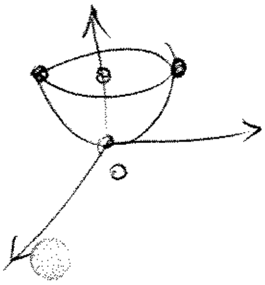
$$= \int_0^2 x^2 \cos x^3 \, dx = \left[\begin{array}{l} x^3 = u \\ 3x^2 dx = du \\ x^2 dx = \frac{1}{3} du \end{array} \right]$$

$$= \frac{1}{3} \int_0^8 \cos u \, du = \frac{1}{3} (+\sin u) \Big|_0^8$$

$$= \frac{1}{3} (\sin 8 - \sin 0)$$

$$= \frac{\sin 8}{3}.$$

6. $$\iiint\limits_K \frac{1}{1+z^2}\,dx\,dy\,dz = \iint\limits_{x^2+y^2\le 1} \int_{x^2+y^2}^{1} \frac{1}{1+z^2}\,dz$$



$$= \iint\limits_{x^2+y^2\le 1} \arctan z \Big|_{x^2+y^2}^{1}\, d\cancel{x}dy$$

$$= \iint\limits_{x^2+y^2\le 1} \left(\arctan 1 - \arctan(x^2+y^2)\right) dx\,dy$$

$$= \frac{\pi}{4}\cdot\pi - \int_0^{2\pi}\int_0^1 \arctan r^2 \cdot r\,dr\,d\theta$$

$$= \frac{\pi^2}{4} - 2\pi\cdot\frac{1}{2}\int_0^1 \arctan u\,du$$

$$= \frac{\pi^2}{4} - \pi\left(u\arctan u\Big|_0^1 - \int_0^1 \frac{u}{1+u^2}\,du\right)$$

$$= \frac{\pi^2}{4} - \frac{\pi^2}{4} + \pi\cdot\frac{1}{2}\ln(1+u^2)\Big|_0^1 = \frac{\pi\ln 2}{2}.$$

7. $F = (x,y,z)$

f ? $\nabla f = F$

$f_1(x,y,z) = x \Rightarrow f(x,y,z) = \frac{x^2}{2} + g(y,z)$

$\left.\begin{array}{l} f_2(x,y,z) = y \\ f_2(x,y,z) = g_1(y,z) \end{array}\right\} \Rightarrow g_1(y,z) = y$

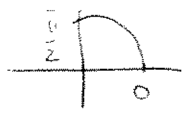
$\Rightarrow g(y,z) = \frac{y^2}{2} + h$

$\left.\begin{array}{l} f_3(x,y,z) = z \\ f_3(x,y,z) = h'(z) \end{array}\right\} \Rightarrow h'(z) = z \Rightarrow$

$h(z) = z^2$

7.
$$\int_{\Gamma} x\,dx + y\,dy + z\,dz = \frac{\pi^2}{2} + \frac{0^2}{2} + \frac{\pi^2}{2} - 0 = \pi^2.$$

8. a)
$$\iint_{Y} F\cdot N\,dS = \iint_{\{x^2+y^2\le 1,\ x,y\ge 0\}} (y\cdot 2x - x\cdot 2y + z\cdot 1)\,dx\,dy$$



$$= \iint_{\{x^2+y^2\le 1\}} (1-x^2-y^2)\,dx\,dy =$$

$$= \int_0^1 \int_0^{\frac{\pi}{2}} (1-r^2)\, r\,dr\,d\theta = \frac{\pi}{2}\left(\frac{r^2}{2} - \frac{r^4}{4}\right)\Big|_0^1$$

$$= \frac{\pi}{2}\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{\pi}{8}.$$

b)
$$\iint_{Y} \frac{1}{\sqrt{2+3x^2+3y^2-z}}\,dS =$$

$$= \iint_{\{x^2+y^2\le 1,\ x,y\ge 0\}} \frac{1}{\sqrt{2+3x^2+3y^2-1+x^2+y^2}}\cdot\sqrt{1+4x^2+4y^2}\,dx\,dy = \frac{\pi}{4}.$$

9.
$$\iint_{x^2+y^2\le 1} f(x^2+y^2)\,dx\,dy = \int_0^1\int_0^{2\pi} f(r^2)\cdot r\,dr\,d\theta =$$

$$= 2\pi\cdot\frac{1}{2}\int_0^1 f(u)\,du = \pi\int_0^1 f(x)\,dx.$$